Context free Grammar's & Language

A Grammer is nothing but a set of rules to define valido sentances in any language, here we are Introducing Context free grammars, Which generate Context free languages. CFL has great practical significance in defining Programming language and in simplyfying the translation for Programming languages.

Context free grammars :-

Mathematicaly Context free grammar is defined as follows

A Grammar $G = (V, T, P, S)$ is said to be Context free.

Where

V = A finite set of Non-Terminal (variable), generally represented by Capital latters $A, B, C, D, \ldots$

T = A finite set of terminal, generally represented by small letters, like $a, b, c, d, e, f, \ldots$

S = Starting non-terminal (variable), Called start symbole of the grammar, $S \in V$

P = Set of rules or Production in CFG.

G is a Context free and all Production in P have the form

$$\alpha \to \beta$$

Where. $\alpha \in V$ & $\beta \in (V \cup T)^*$

Note:- Every Regular Grammar is Context free, so a regular language is also a Context free one, It is already Proved by Pumping lemma that language $\{a^n b^n \mid n \geq 0\}$ is not regular but it is Possible to design a CFG for these Language.

"So it is very clear that "The family of regular language is a Proper subset of the family of CFL""

~~Eg1: Consider a grammar $G = (V, T, P, S)$ where~~

(Lecture 2,03)

Q.1. Construct a CFG for language $L = \{a^n b^n \mid n \geq 1\}$

Ans: $G = (V, T, P, S)$

$V = \{S\}$

$T = \{a, b\}$

$P \rightarrow \quad S \rightarrow aSb \mid ab$

Q.2. Construct a CFG for language $WW^R \mid W \in (a,b)^*$

Ans. $G = (V, T, P, S)$

$V = \{S\}$

$T = \{a, b\}$

$P \rightarrow$

$S \rightarrow aSa \mid bSb \mid a \mid b \mid \epsilon$

Q.3. Write a CFG, which generate palindrome for binary no.

Ans: $G = (V, T, P, S)$

$V = \{S\}$

$T = \{0, 1\}$

$P \rightarrow$

$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$

Q. Design a CFG for the language $L = \{a^n b^m : n \neq m\}$

Ans:

if $n \neq m$ then there are only two cases are possible

<u>Case. 1.</u> if $n > m$

$L_1 = \{a^n b^m : n > m\}$

$G_1 = (V_1, T_1, P_1, S_1)$

$V_1 = \{S_1, A, S_1'\}$

$T_1 = \{a, b\}$

P →

$S_1 \rightarrow AS_1'$

$S_1' \rightarrow aS_1'b \mid \epsilon$

$A \rightarrow aA \mid a$

Case 2: if $n < m$

$L_2 = \{a^n b^m : n > m\}$

$G_2 = (V_2, T_2, P_2, S_2)$

$V_2 = \{S_2, B, S_2'\}$

$S_2 = S_2' B$

$S_2' = aS_2'b \mid \epsilon$

$B \rightarrow bB \mid b$

final Grammar for language L

$S \rightarrow S_1 \mid S_2$

Q.4. Construct a CFG for set of all string of length 2 for $\Sigma = \{a, b\}$

Ans.

$G = (V, T, P, S)$

$V = \{S, A\}$

$T = \{a, b\}$

$P \rightarrow$

$S \rightarrow AA$

$A \rightarrow a \mid b \mid$

Q.5. Construct the CFG for the language $L = \{WcW^R \mid W \in (a, b)^*\}$

Ans.

$G = (V, T, P, S)$

$V = \{S\}$

$T = \{a, b\}$

$P \rightarrow$ $S \rightarrow aSa \mid bSb \mid c$

PARSE Tree: (Derivation Tree):

There is a Tree representation for derivation that has proved extremely useful.

A Parse Tree is an ordered tree in which nodes are labeled with the left side of productions and in which the children of a node represent it's corresponding right side.

Definition:

Let $G = (V, T, P, S)$ be a CFG. An ordered tree for this CFG, G is a derivation tree if and only if it has the following properties.

(a) The root is labeled by the starting non-terminal of the CFG that is S

(b) Every leaf of the ordered tree has a label from $T \cup \{\epsilon\}$.

(c) Every Interior node of ordered tree has a label from **V**.

(d) Let us assume that a vertex has label $X \in V$, and it's children are labeled (from left to right) $Y_1, Y_2, Y_3, \ldots Y_n$. Then Production must contain a production of the form

$$X \rightarrow Y_1, Y_2 \ldots Y_n$$

e) A leaf labeled ϵ has no siblings, that is, vertex with a child labeled ϵ can have no other children. Clearly if the leaf is labeled ϵ, then it must be the only child of it's parent.

The yield of Parse Tree:

If we look at the levels of any Parse tree and can concate them from the left, we get a string, called the yield of the tree. Which is always a string that is derived from the root will be proved shortly of special importance are those parse tree such that

(a) The yield is a Terminal string. All leaves are labeled either with a terminal or with ϵ.

(b) The root is labeled by the start symbole.

Exp.1. Consider the CFG

$$S \to XX$$

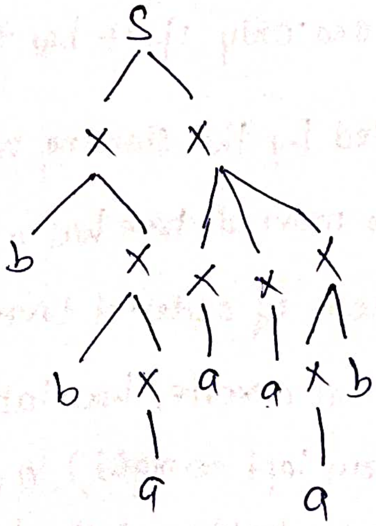
$$X \to XXX \mid bX \mid Xb \mid a$$

find the Parse tree for the string bbaaaab

Sol.

$$S \to XX$$

$$X \to XXX \mid bX \mid Xb \mid a$$



Q.2 Consider the grammar G, with Production

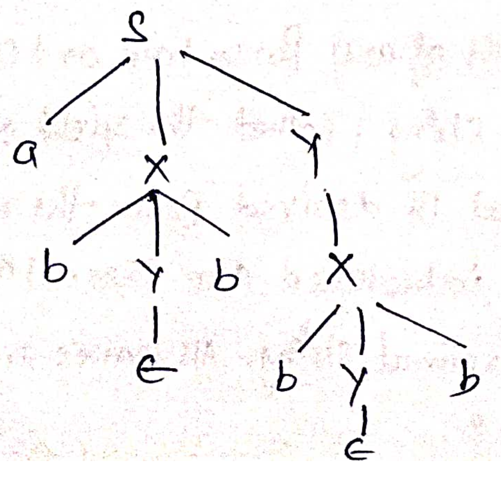
$$S \to aXY$$

$$X \to bYb$$

$$Y \to X \mid \epsilon$$

find the Parse tree for the string abbbb

Sol.



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Q3: Write a CFG for a language $L = \{x0^n y1^n z \mid n >= 0\}$ and give the Parse tree for the string x000y111z

Ans:

CFG: $G(V, T, P, S)$

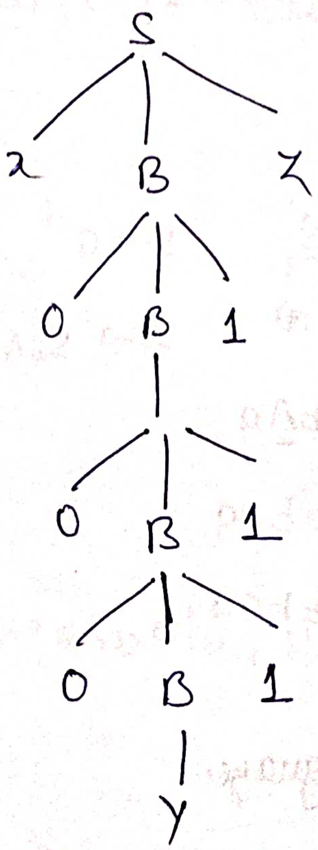
$V = \{S, \quad\}$

$T = \{x, y, z, 0, 1\}$

$P \rightarrow$

$S \rightarrow xBz$

$B \rightarrow 0B1 \mid y$



x000y111z

Q.4. Consider the CFG for the string aabbaa

$S \to aAS \mid a$

$A \to SbA \mid ba$

Leftmost

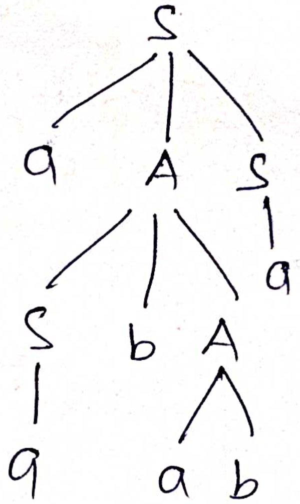
$S \to a\underline{A}S$ ~~S→aAS~~ $A \to SbA$

$a\underline{S}bAS$ $S \to a$

$aabAS$ $A \to ba$

$aabbaS$ $S \to a$

$aabbaa$



Right most:

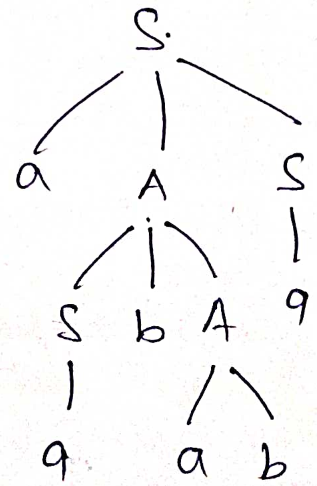
$S \to aA\underline{S}$ $S \to a$

$\to a\underline{A}a$ $A \to SbA$

$\to aSb\underline{A}a$

$\to aSbbaa$

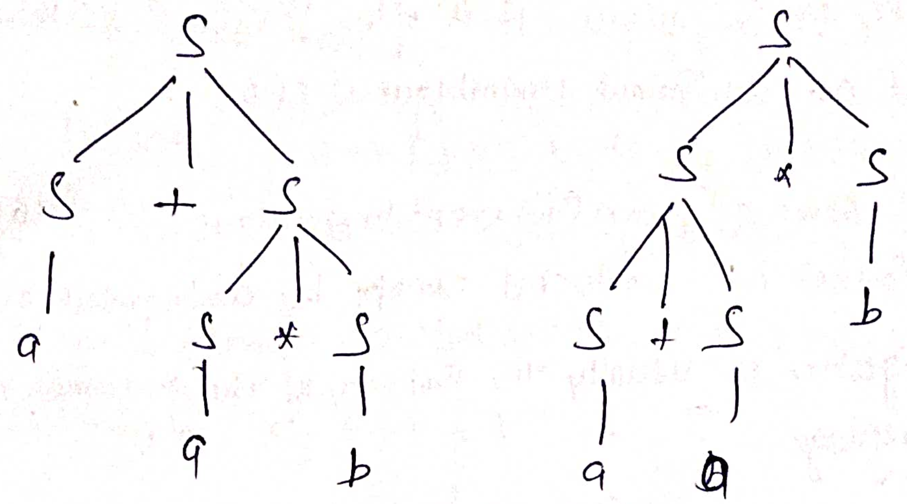
$\to aabbaa$



Ambiguity in grammars & language:

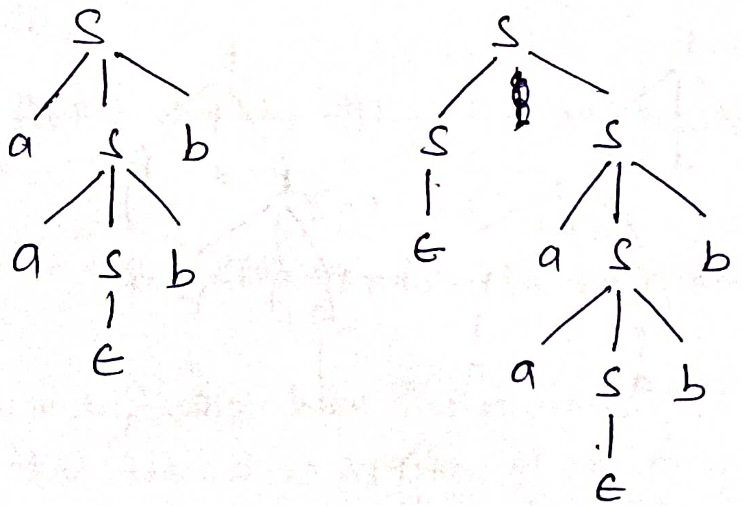
A CFG is called ambigious if for at lest one word in the language that is generates there are two possible derivations of the word that correspond to diffrent syntax tree. if CFG is not ambiguous. it is colled unambiguous.

Exp. If CFG is $S \to S+S \mid S*S \mid a \mid b$, show that G is ambiguous.



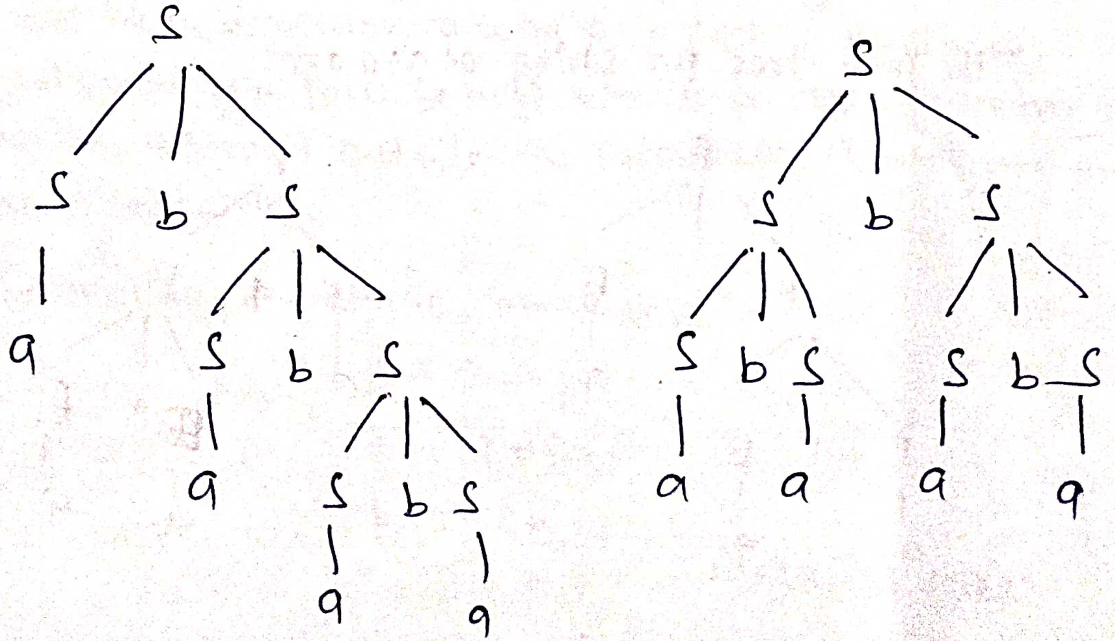
Exp. If CFG is $S \to aSb \mid SS \mid \epsilon$, show that G is ambiguous for aabb.

Ans.



Exp. If CFG G is $S \to aSbS \mid a$, show that G is ambiguous for abababa.

Ans



Removing Ambiguity for Grammars!.

* If a CFG is imbiguous. it is often Possible & usually desirable to find an equivalent unambiguous CFG

* Although some CFG are ("inherently ambiguous" in the sence that they cannot be produced except by ambiguous grammar.

* Ambiguity is usually the Property of the grammar rather than the language.

Let's Consider the Grammar for algebric expression and for the string a+a*a the two parse tree are possible

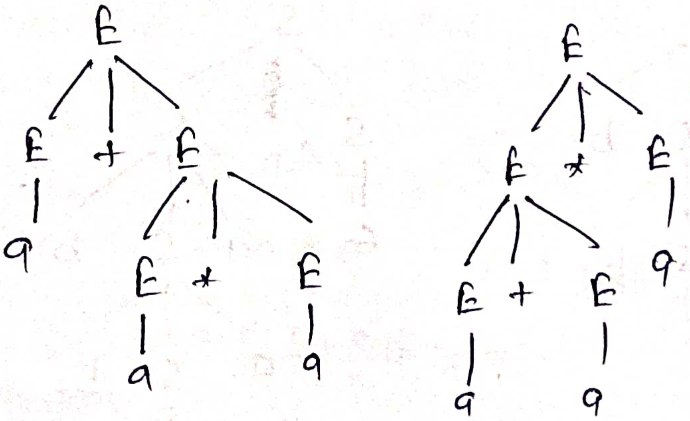
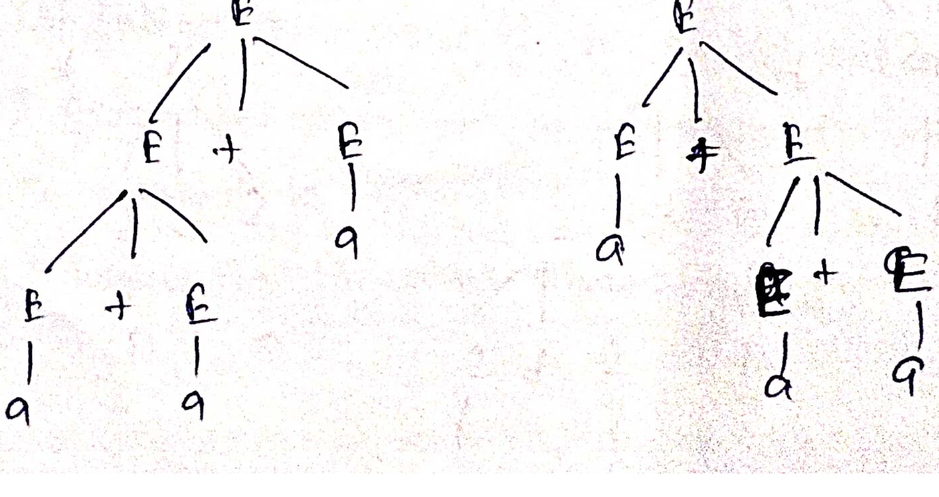


fig-1.

In the above grammar the Precedence of the operator + & * is the cause of ambiguity.

The Parse tree for string a+a+a are



The Cause of ambiguity in String a+a+a is the associativity of + operator. If we assume the + operator is left associative the String a+a+a is equivalent to (a+a)+a.

* We see $E \rightarrow E+E$, by it self is enough to produce ambiguity. so we need the need to sum of something.

* The word "terms" to stad for this things that are added.
  the Grammar will be, $E \rightarrow E+T \mid T$

* The Expression Can also be products; like a+b*c and a*b+c are both sums, so it is right to says that term Can be product.

* The factor are the things that are multiplied to produce terms

$$T \rightarrow T * F \mid F.$$

The corresponding variable in the grammar will be F

* Since the Multiplication have higher precedence over addition, we can say that Expression are sum of one or more terms, & terms are products of one or more factors.

* Now, we Must deal with Parentheses, we might say that (E) Could be an expression, a term or a factor. However, evaluation of a Parenthetical expression takes precedence over any operator outside the parentheses. Factor are evaluated first in hierarchy..

The resulting unambiguous grammar is

$$E \rightarrow E+T \mid T$$
$$T \rightarrow T*F \mid F$$
$$F \rightarrow (E) \mid a$$

identifier (id)

Simplified CFG and It's Normal forms

Reduction of CFG:

There are several ways in which one can restrict the format of CFG without reducing the language generation power. Let L be a CFL, then it can be generated by a CFG G with the following properties.

(a) We must eliminate useless symbole. those variable or terminals that do not appears in any derivation of a terminal string from the start symbol.

(b) We must eliminate ε-productions, those of the form $X \to \epsilon$ for some variable X.

(c) We must eliminate unit production, those of the form $X \to Y$ for variable X and Y

1. Eliminate Useless Symbole!.

A symbol that is useful will be both generating and rechable. If we eliminate the symbole that are not generating first, and then eliminate from the remaining grammar those symbols that are not rechable. then after this process, CFG will have only useful symbols.

There for reduction of a given grammar G, involves following steps:

(a) Identified non-generating symbols in given CFG and eliminate those productions which contain non-generating symbols.

(b) Identified non-rechable symbols in grammar and eliminate those productions which contains non-rechable symbols.

Exp. Consider a CFG

$$S \to AB \mid a$$
$$A \to b$$

· identified & Eliminate useless symbole

P.T.O.

Ans.

$S \to AB \mid a$

$A \to b$

Here B is non-generating so eleminate the production of B

$S \to a$

$A \to b$

Now A is not rechable the eleminate it

$S \to a$

Q. Remove the useless symbols from the given CFG

$S \to aB \mid bX$

$A \to BAd \mid bSX \mid a$

$B \to aSB \mid bBX$

$X \to SBD \mid aBx \mid ad$

Ans:- B is not generation symbol. It does not produce terminal in W(t) and A is Not rechable to remove both A & B from G.

$S \to bX$

$X \to ad$

Q. Consider the following grammar & obtain an equivalent grammar containing no useless grammar symbol.

$A \to xyz \mid Xyzz$

$X \to Xz \mid xYz$

$Y \to yYy \mid Xz$

$Z \to Zy \mid z$

Ans. $A \to xyz$ and ~~xxxx~~ $Z \to z$ directely deriving to the string of terminal so it is usefull symbols & X, Y do not lead to a string of terminal so remove X & Y production from G.

$A \rightarrow xyz$

$Z \rightarrow Zy \mid z$

Now A is starting Non-terminal and in Production of A does not Contain Z, so Z is not rechable, so remove it from G.

$A \rightarrow xyz$

Q. Find the reduced grammar that is equivalent to the CFG given below

$S \rightarrow aC \mid SB$

$A \rightarrow bSCa$

$B \rightarrow aSB \mid bBC$

$C \rightarrow aBC \mid ad$

Ans: $C \rightarrow ad$, therefore, C is generating symbole, B is useless do not generate terminal and A is not rechable from start variable S. So Eliminate A & B

$S \rightarrow aC$

$C \rightarrow ad$

Removal of Unit Production:- A Production in the form of

Non-terminal $\rightarrow$ One-Non-terminal

That is the Production in the form of $A \rightarrow B$ is called unit Production.

Exp. Consider the CFG.

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow C \mid b$

$C \rightarrow D$

$D \rightarrow E$

$E \rightarrow a$

Ans. In above Production three unit Productions

$B \rightarrow C$

$C \rightarrow D$

$D \rightarrow E$

then resolving unit productions.

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow C \mid b$
$C \rightarrow a$
$D \rightarrow a$
$E \rightarrow a$

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow a \mid b$
$C \rightarrow a$
$D \rightarrow a$
$E \rightarrow a$

Now C, D, E is Not rechable Production or useless Production

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow a \mid b$

Q. Consider the following unambiguous expression grammar

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$F \rightarrow I \mid (E)$
$T \rightarrow F \mid T * F$
$E \rightarrow T \mid E + T$

Ans). There are three unit Productions.

$T \rightarrow F$
$F \rightarrow I$
$E \rightarrow T$

$F \rightarrow I$ remove by.

$I \rightarrow a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$F \rightarrow (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$
$T \rightarrow F \mid T * F$
$E \rightarrow T \mid E + T$

Now we eliminate $T \to F$ with the help of

$$F \to (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$

Now Grammar

$$I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$F \to (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$T \to T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$E \to T \mid E+T$$

Now let us remove $E \to T$ with the help of

$$T \to T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid Ia \mid I1$$

New Grammar is

$$I \to a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$F \to (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$T = T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$
$$E \to E+T \mid T*F \mid (E) \mid a \mid b \mid Ia \mid Ib \mid I0 \mid I1$$

Removal of ∈-Productions:-

- Now for our attention to the eliminate the production of the form $A \to \epsilon$ which is called ∈-Production.
- If ∈ is in L(G), we can not eliminate all ∈-Production from G, but if ∈ is not in L(G), we can eliminate all ∈-Productions from G.

To eliminate ∈-Productions from a grammar G we can use the following Technique:.

If $A \to \epsilon$ is a Production to be eliminate then we look for all productions, whose right side contain A, and replace each occurance of A in each of these Productions to obtain the non ∈-Production.

R.R.V.

Q. Consider the following grammar.

$S \to aA$

$A \to b \mid \epsilon$

Ans!- Put the ϵ in place of A in right side and new production added.

$S \to aA \mid a$

$A \to b$

Q- Consider the following grammar G.

$S \to ABAC$

$A \to aA \mid \epsilon$

$B \to bB \mid \epsilon$

$C \to c$

Ans!. Put the value of A & B $\to \epsilon$ to the right side the new Production's are.

$S \to ABAC \mid BAC \mid ABC \mid BC \mid AAC \mid AC \mid C$

$A \to aA \mid a$

$B \to bB \mid b$

$C \to c$

Q. Consider the following grammar & remove ϵ-Productions

$S \to aSa$

$S \to bSb \mid \epsilon$

Ans.

$S \to aSa \mid bSb \mid aa \mid bb$

1. CHOMSKY Normal form:-

If a CFG has only production of the form

$A \to BC$
$A \to a$

is said to be chomsky normal form or CNF.

Q. Change the following Grammar $G = \{\{S, A, B\}, \{a, b\}, P, S\}$

$S \to bA | aB$
$A \to bAA | aS | a$
$B \to aBB | bS | a$

in to CNF.

Ans:

CNF follow the property of production is $A \to BC$, $A \to a$.
Some of the production do not follow the property of CNF.
then we simplify it through adding some production

$S \to X_bA \mid X_aB$
$A \to X_bAA \mid X_aS \mid a$
$B \to X_aBB \mid X_bS \mid a$
$X_a \to a$
$X_b \to b$

Production $A \to X_aAA$ and $B \to X_aBB$ do not follow the property of CNF, $A \to BC$, so

~~$A \to X_aAA$~~

$B \to X_aD$	$C \to AA$
$A \to X_bC$	$D \to BB$

New Grammar

$S \to X_aA \mid X_aB$
$A \to X_bD \mid X_aS \mid a$
$B \to X_aD \mid X_bS \mid a$
$X_a \to a$
$X_b \to b$
$C \to AA$
$D \to BB$

Q. Change the following grammar into CNF.

$S \rightarrow 1A \mid 0B$

$A \rightarrow 1AA \mid 0S \mid 0$

$B \rightarrow 0BB \mid 1$

Ans: Replace AA by C_a, and BB by C_b, and 1 by C_1, 0 by C_0

$S \rightarrow C_1A \mid C_0B$

$A \rightarrow C_1AA \mid C_0S \mid 0$

$B \rightarrow C_0C_b \mid 1$

$A \rightarrow C_1AA$ do not follow the CNF Property.

$S \rightarrow C_1A \mid C_0B$

$A \rightarrow C_1C_a \mid C_0S \mid 0$

$B \rightarrow C_0C_b \mid 1$

$C_a \rightarrow AA$

$C_b \rightarrow BB$

$C_0 \rightarrow 0$

$C_1 \rightarrow 1$

Q. Chang the following grammar into CNF

$G = (\{S\}, \{a, b, c\}, \{S \rightarrow a \mid b \mid cSS\}, S)$

Ans:

$S \rightarrow a \mid b \mid cSS$

Replace SS by D in Production

$S \rightarrow a \mid b \mid cD$

$D \rightarrow SS$

Q. Chang the following grammar into CNF

$$S \to abSb \mid a \mid aAb$$
$$A \to bS \mid aAAb$$

Ans:

Replace aA by B_a and Ab by B_b

$$S \to abSb \mid a \mid B_ab$$
$$A \to bS \mid B_aB_b$$
$$B_a \to aA$$
$$B_b \to Ab$$

Replace ab by C and Sb by D

$$S \to CD \mid a \mid B_ab$$
$$A \to bS \mid B_aB_b$$
$$B_a \to aA$$
$$B_b \to Ab$$
$$C \to ab$$
$$D \to Sb$$

Replace a by X_a and b by X_b

$$S \to CD \mid a \mid B_aX_b$$
$$A \to X_bS \mid B_aB_b$$
$$B_a \to aA$$
$$B_b \to Ab$$
$$C \to X_aX_b$$
$$D \to SX_b$$
$$X_a \to a$$
$$X_b \to b$$

Q. Convert CFG which is given below into CNF form

$$S \to bA \mid aB$$
$$A \to bAA \mid aS \mid a$$
$$B \to aBB \mid bS \mid b$$

Sol.

Replace b by X_b and a by X_a

$$S \to X_bA \mid X_aB$$
$$A \to X_bAA \mid X_aS \mid a$$
$$B \to X_aBB \mid X_bS \mid b$$
$$X_a \to a$$
$$X_b \to b$$

~~Replace $X_aA \to C_a$ and $X_bB \to C_b$~~

$$\cancel{S \to X_bA \mid X_aB}$$
$$\cancel{A \to X_aA \mid X_aS \mid a}$$
$$\cancel{B \to X_aC_b \mid X_bS \mid b}$$
$$\cancel{X_a \to a}$$
$$\cancel{X_b \to b}$$
$$\cancel{C_a \to AA}$$
$$\cancel{C_b \to BB}$$

Replace $X_bA \to C_a$ and $X_aB \to C_b$

$$S \to X_bA \mid X_aB$$
$$A \to C_aA \mid X_aS \mid a$$
$$B \to C_bB \mid X_bS \mid b$$
$$C_a \to X_bA$$
$$C_b \to X_aB$$
$$X_a \to a$$
$$X_b \to b$$

Q. Design a CFG for the language $L = \{a^{4n} \mid n \geq 1\}$ and Convert that CFG into CNF form.

Ans

Let CFG for the language $L = \{a^{4n} \mid n \geq 1\}$ is G.

$G = (V, T, P, S)$

$V = \{S\}$

$T = \{a\}$

P→

$S \rightarrow aaaaS \mid aaaa$

Now let us Convert this CFG into CNF ~~form~~ form. replace each a by A then CFG

$S \rightarrow AAAAS \mid AAAA$

$A \rightarrow a$

the CFG form Will be.

$S \rightarrow AR_1$

$R_1 \rightarrow AR_2$

$R_2 \rightarrow AR_3$

$R_3 \rightarrow AS$

$S \rightarrow AR_4$

$R_4 \rightarrow AR_5$

$R_5 \rightarrow AA$

$A \rightarrow a$

(Lecture-07)

GREIBACH NORMAL FORM :-

For Every Context free language L without ϵ, there exist a grammar in which every production is of the form $A \to aV/a$, where 'A' is a variable, 'a' is exactly one terminal and 'V' is the string of none or more variable, clearly $V \in V_n^*$

Q. Convert the grammar

$$S \to AB \mid BC$$
$$A \to aB \mid bA \mid a$$
$$B \to bB \mid cC \mid b$$
$$C \to c$$

into GNF

Ans. Production $S \to AB \mid BC$ is not in GNF. On applying the substitution rule we got the grammar.

$$S \to aBB \mid bAB \mid aB \mid bBC \mid cCC \mid bC$$

in GNF.

Exp. Convert the grammar

~~$S \to aBASA \mid aBA$~~

$$S \to abaSa \mid aba$$

Ans. If we introduce new variable A and B and productions as $A \to a$ & $B \to b$ & Substitute into given grammar as.

$$S \to aBASA \mid aBA$$
$$A \to a$$
$$B \to b$$

Q... Convert the grammar

$$S \rightarrow AB$$
$$A \rightarrow BS \mid a$$
$$B \rightarrow SA \mid b$$

into GNF

Ans:. The given grammar is in CNF, let us change the name of variable as A_1, A_2, A_3 for S, A, B respectively, Now the given productions are in the form (with A_1 as start symbol)

$$A_1 \rightarrow A_2 A_3$$
$$A_2 \rightarrow A_3 A_1 \mid a$$
$$A_3 \rightarrow A_1 A_2 \mid b$$

In the above grammar we need production must start with terminal or with higher no. variable.

Applying substitution rule for A_1 then we get

$$A_3 \rightarrow A_2 A_3 A_2 \mid b$$

Applying substitution rule for A_2

$$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid a A_3 A_2 \mid b$$

Eliminating the left recursion from A_3 production, we get

$$A_3 \rightarrow A_3 B_1 \mid a A_3 A_2 \mid b$$

$$B_1 \rightarrow A_1 A_3 A_2 B_1 \mid A_1 A_3 A_2$$

$$A_3 \rightarrow a A_3 A_2 B_1 \mid b B_1 \mid a A_3 A_2 \mid b$$

B_1 is new variable introduced.

P.T.O

Now all the A_3 Production start with terminals. With the help of substitution rule we can replace A_3 in R.H.S of A_2 production.

$$\boxed{A_2 \rightarrow aA_3A_2B_1A_1 \mid bB_1A_1 \mid aA_3A_2A_1 \mid bA_1 \mid a}$$

Now, all A_2 Productions start with terminal. Again we replace A_2 from $A_1 \rightarrow A_2A_3$

$$\boxed{A_1 \rightarrow aA_3A_2B_1A_1A_3 \mid bB_1A_1A_3 \mid aA_3A_2A_1A_3 \mid bA_1A_3 \mid aA_3}$$

one Production $B_1 \rightarrow A_1A_3A_2B_1 \mid A_1A_3A_2$ is not in GNF. Now applying substitution rule in production.

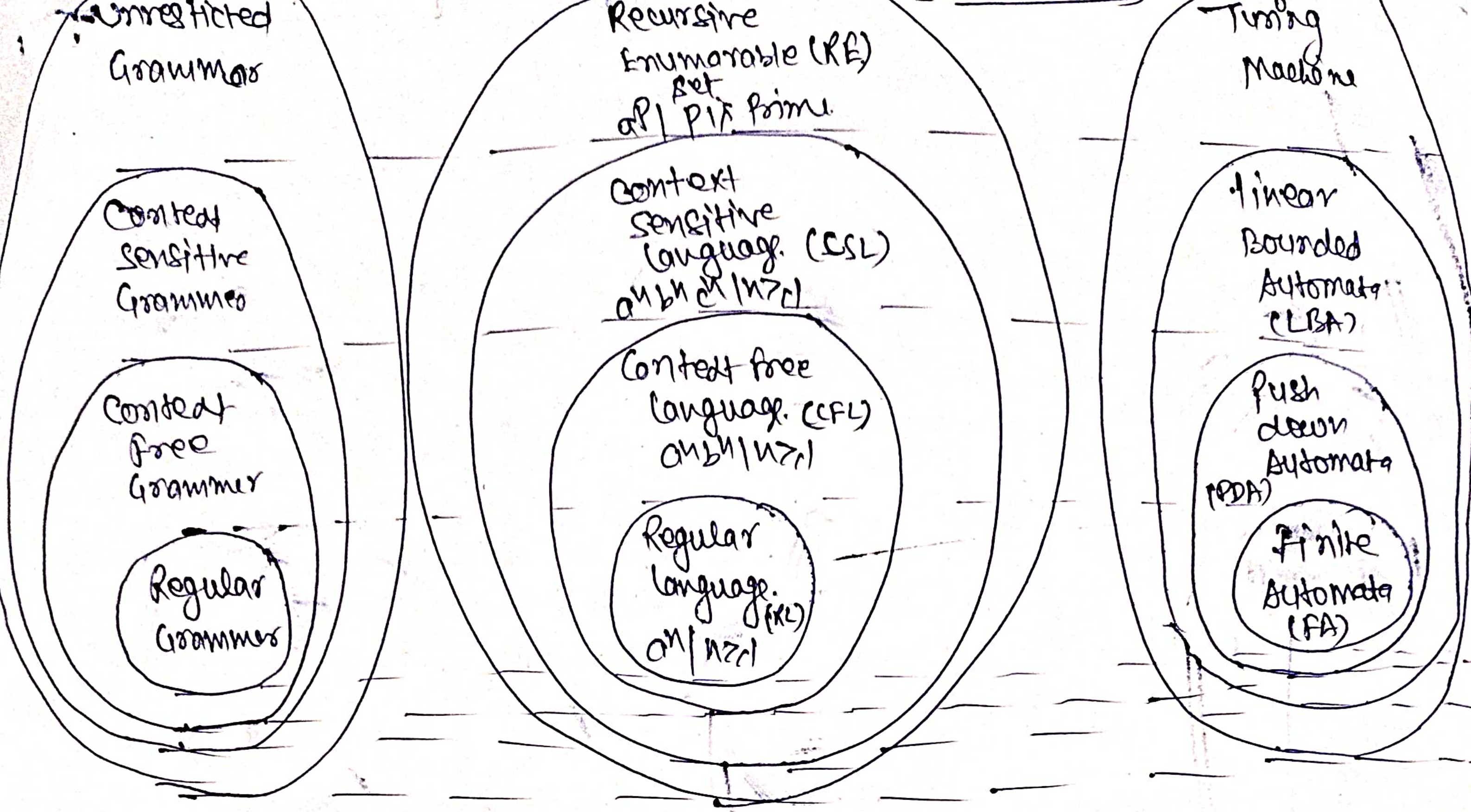
$$B_1 \rightarrow aA_3A_2B_1A_1A_3A_3A_2B_1 \mid bB_1A_1A_3A_3A_2B_1 \mid aA_3A_2A_1A_3A_3A_2B_1 \mid bA_1A_3A_3A_2B_1 \mid aA_3A_3A_2B_1 \mid aA_3A_2B_1A_1A_3A_3A_2 \mid bB_1A_1A_3A_3A_2 \mid aA_3A_2A_1A_3A_3A_2 \mid bA_1A_3A_3A_2 \mid aA_3A_3A_2$$

All Productions are.

$$A_1 \rightarrow aA_3A_2B_3A_1A_3 \mid bB_3A_1A_3 \mid aA_3A_2A_1A_3 \mid bA_1A_3 \mid aA_3$$
$$A_2 \rightarrow aA_3A_2B_3A_1 \mid aB_3A_1 \mid aA_3A_2A_1 \mid bA_1 \mid a$$
$$A_3 \rightarrow aA_3A_2B_1 \mid bB_1 \mid aA_3A_2 \mid b$$
$$B_1 \rightarrow aA_3A_2B_1A_1A_3A_2A_3B_1 \mid bB_1A_1A_3A_2A_3B_1 \mid aA_3A_2A_1A_3A_3A_2B_1 \mid bA_1A_3A_2B_1 \mid aA_3A_3A_2B_1 \mid aA_3A_2B_1A_1A_3A_3A_2 \mid bB_1A_3A_3A_2 \mid aA_3A_2A_1A_3A_2 \mid bA_1A_3A_3A_2 \mid aA_3A_3A_2$$

Ans

{Lecture-08}



CHOMSKY HIERARCHY

UNIT III: (Important Questions)

1. Consider the grammar G given as follows : $S \rightarrow AB \mid aaB$ $A \rightarrow a \mid Aa$ $B \rightarrow b$, Determine whether the grammar G is ambiguous or not. If G is ambiguous then construct an unambiguous grammar equivalent to G.
2. The family of context free languages is closed under star-closure but is not closed under difference.
3. Given a context free Grammar G. Write an algorithm (if it exists) to determine whether L(G) is infinite or not.
4. Given the following CFG having S as start symbol, find an equivalent CFG with no useless symbols : $S \rightarrow aAa \mid bSb \mid \in$ $A \rightarrow C \mid a$ $B \rightarrow C \mid b$ $C \rightarrow CDE \mid \in$ $D \rightarrow A \mid B \mid ab$.
5. DesigntheCFGforthefollowinglanguage: (UPTU 2018-19)
   i) $L=\{0^m1^n \mid m \neq n \;\&m,n \geq 1\}$
   ii)$L= \{a^lb^mc^n \mid l+ m= n \;\&l,m\geq 1\}$

ProvethatthefollowingLanguageL=$\{a^nb^nc^n\}$isnotContextFree

6. What is difference between Chomsky normal form (CNF) and Greibach normal form (GNF) ? Convert the following grammar in Greibach normal form : $S \rightarrow AB$ $A \rightarrow BSB \mid BB \mid b$ $B \rightarrow a \mid aAb$.(UPTU 2013-14)
7. Find a Context Free Grammar (CFG) generating the following language : $L = \{a^ib^jc^k \mid i = j \text{ or } i = k\}$.
8. Describe the language generated by the following grammar : $S \rightarrow bS/aA/\varepsilon$ $A \rightarrow aA/bB/b$ $B \rightarrow bS$.
9. Show that the given grammar is ambiguous. Also find an equivalent unambiguous grammar. $S \rightarrow ABA$ $A \rightarrow aA/\varepsilon$ $B \rightarrow bB/\varepsilon$.(UPTU 2013-14)
10. Given context free Grammar, how do you determine that grammar as : (i) Empty or Non Empty (ii) Finite or Non-Finite (iii) Whether a string x belongs to language of grammar.(UPTU 2012-13)
11. Design a CFG for the language consisting of all strings of even length over {a,b}.